Empirical Study on Theoretical Option Pricing Models

Dr. Shailesh Rastogi*
Nithya Vetriselvam**
Jeffrey Don Davidson***

Abstract

An option is a contract giving the buyer the right, but not the obligation, to buy (call option) or sell (put option) an underlying asset (a stock or index) at a specific price on or before a certain date. An option is a derivative. In the case of a stock option, its value is based on the underlying stock (equity) and if it is an index option, its value is based on the underlying index. An option is a security, just like a stock or bond, and constitutes a binding contract with strictly defined terms and properties. Some people remain puzzled by options. The truth is that most people have been using options for some time, because optionality is built into everything from mortgages to auto insurance. Several pricing models like Black-Scholes Model, Binomial Option Pricing Model, Stochastic volatility models etc., have been developed over the years to calculate the price of the options. The main objective of this paper is to test the consistency of these models by calculating the prices of the options for 175 companies listed in the National Stock Exchange (NSE) using Black – Scholes model and Binomial Tree pricing model, and comparing it with the current market option price. The study also checks the option pricing models for no arbitrage conditions such as the put-call parity using statistical methods.

Keywords: *Options, Call options, Put options, Black-Scholes Model, Binomial Option Pricing Model, put-call parity.*

Introduction

Options are considered by many individuals as a new form of speculative instrument when contrasted with other more customary structures, for example, stocks and bonds. But the fundamental idea of options is said to be originated from Ancient Greece. The use of options for speculation was first recorded by the Greek philosopher Aristotle in 4th century BC. In his book "Politics", he mentions the use of call options by an astronomer named Thales who takes advantage of the increase in olive harvests which Thales had earlier predicted. Though the terminology came later, the structure of the payoff resembles the call options of today.

Another prominent example of the use of options was in Holland at the time of 17th century for trading of tulip flowers. The demand for tulip flowers began to increase at that time. So traders thought it would be an opportunity to bet that the price of tulip flowers will increase and started to speculate. As the price went up, the value of these contracts also went up. More and more traders continued to speculate on the increase of the price and soon a bubble was created. Like every bubble, this bubble also burst. A lot of people lost their money and some others could not meet their obligations. As a result, the country was thrown into recession.

An outstanding improvement in the historical backdrop of options trading was facilitated by an American lender by the name of Russell Sage. In the late nineteenth century, Sage started making call and put options that could be exchanged over the counter in the United States. Sage is also credited to be the first individual to set up a relationship between the option price, the price of the underlying, and the rate of interest. He also used the method of put-call parity as a way to understand the interest rate that can be charged against the options.

Though various changes took place in the options market, there was no stimulus to regulate it until the 1960's when Securities and Exchange Commission (SEC)

*Dean-MBA, Thiagarajar School of Management, Madurai, e-mail: drshailrastogi@gmail.com
**Assistant Systems Engineer, TCS, Chennai
***MBA Final Year Student, Thiagarajar School of Management, Madurai

stepped in. In 1973 the CBOE established a market place to trade and the options clearing corporation was established to prevent counterpart risk. However there was still lack of participation as majority of investors were not aware of ways to evaluate the option prices. However another revolutionary finding took place in the same year which changed everything about option pricing.

In 1973, two professors namely Fischer Black and Myron Scholes developed a model that could be used to calculate the price of an option. This came to be known as the Black-Scholes Option Pricing Model otherwise known as the Continuous Option Pricing Model. Now, traders had a way of understanding the option pricing mechanism and take advantage of any underpricing or overpricing in the market.

Many models have been developed over the course of time but the commonly used models are the Continuous Option Pricing Model and the Binomial Option Pricing Model. However, do the models reflect the prices of the market even today? Or the market has begun to overstate the prices of the options which cannot be completely explained by the theoretical models. This study attempts to answer the questions by comparing the market price of the options of 175 companies listed in the National Stock Exchange with the prices estimated using theoretical models and also by checking for no arbitrage conditions such as put-call parity using statistical tools.

Review of Literature

In last two decades, option pricing has witnessed an explosion of new models that each relaxes some of the restrictive Black-Scholes assumptions. So it is clear that the pricing of options started from evolution of Black-Scholes model. Every option pricing model has to make three basic assumptions: the underlying price process, the interest rate process and the market price of factor risks. For each of the assumption, there are many possible choices. For instance, the underlying price can follow either a continuous- time or discrete-time process. For the term structure of interest rates, there are similarly many choices. So the search for that perfect option pricing model can be endless.

As a practical matter, that perfectly specified option pricing model is bound to be too complex for applications. Ultimately, it is a choice among misspecified models, made perhaps based on

i. Which is the least misspecified?
ii. Which results in the lowest pricing errors?
iii. Which achieves the best hedging performance?

These empirical questions must be answered before the potential of recent advances in theory can be fully realized in practical applications (Gurdip Bakshi & Zhiweu Chen 1997). Besides the obvious normative reasons, a common motivation for these models is abundant empirical evidence that the benchmark BS formula exhibits strong pricing biases across both moneyness and maturity and that it especially under prices deep out-of-of-the-money puts and calls(Bates 1996).

Since Black-Scholes is the starting point of option pricing model, it would be relevant to study the consistency of it with the any other model and also evaluating consistency both models using put-call parity. In Black- Scholes model, the stock price distribution is assumed to be log normal and the volatility of the stock price (σ) and the risk free rate (r) are assumed to be constant. Due to restrictive assumptions, they are known as emprirical basis across moneyness and term structure in Black-Scholes model (Derman & Kani 1994; Dupire 1994; Rubinstein, 1985; Rubinstein 1994). Black-Scholes solved their differential equation to obtain formulas for the prices of European Call and put options (Black- Scholes 1973; Hull 1997). The analytical prices of European call and the boundary condition is

$$c = SN(d_1) - Ke^{-rt} N(d_2)$$

The analytical prices of European put is

$$p = Ke^{-rt} N(-d_2) - SN(-d_1)$$

Where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) * T}{\sigma\sqrt{t}}$$

And

$$d_2 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right) * t}{\sigma\sqrt{t}} = d_1 - \sigma\sqrt{t}$$

N(d1) and N(d2) are the cumulative density functions of the standard normal distribution and t is the time to maturity in years. There are handful of empirical studies that focus on demonstrating the techniques used to calculate the option prices. Chen (1975) uses dynamic programming to price options. He concludes that this technique accurately prices options if reasonable estimates of the expected return and volatility of the underlying stock can be obtained. Norrren and Wolfson (1981) uses total of 52 observations of option prices to test a Black-Scholes model that assumes the underlying stock follows a log normal diffusion process and a model that assumes the stock prices follows a elasticity of Variance diffusion process. Schwartz (1977) uses a different approach to approximate solutions to the differential equation that describes the option pricing values. He examines only 17 observations of AT&T option prices as the focus of the paper is demonstrating the finite differences between the theoretical option prices and the actual market price of those options. Beni and Paul (1990) have estimated over 25000 option prices to empirically investigate the potential problems with the commonly used pricing model Black-Scoles and Binomial tree pricing model. The most important problem is the variance assumption of the stocks taken for analysis.

The second method that we are going to analyze the consistency is Binomial Pricing Model. Binomial Pricing model is used when assumption is made that the underlying option price follows a process where at a given times the price can jump either up or down.The original source of binomial option pricing was pioneering work on binomial trees (Cox, Ross & Rubinstein 1979).

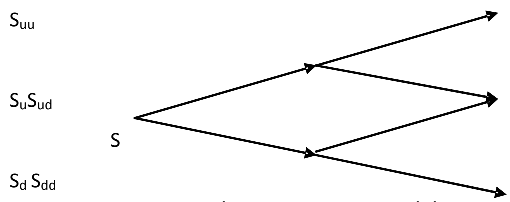

Fig: 1: Binomial Option Pricing Model

The terminal distribution of the price of the underlying asset can be approximated at the maturity date of the option. At each node of this tree, the value of the option can be found under no arbitrage assumption and the principle of risk- neutral valuation (Tian & Yisong 1999).Valuation is performed iteratively, starting at each of the final nodes (those that may be reached at the time of expiration), and then working backwards through the tree towards the first node (valuation date). The value computed at each stage is the value of the option at that point in time.

Option valuation using this method is, as described, a three-step process:

i. Price tree generation
ii. Calculation of option value at each final node
iii. Sequential calculation of the option value at each preceding node

Given stock price S, strike price K, time to maturity t, volatility σ, length of one time step ΔT, and the number of time period n, the set of equations defining the tree is then

$$\text{LengthofonetimeStep}\Delta T = \frac{T}{n}$$

$$\text{Uptickrateu} = e^{\sigma\sqrt{\Delta T}}$$

$$\text{DowntickRated} = \frac{1}{u} = e^{-\sigma\sqrt{\Delta T}}$$

$$\text{RiskneutalProbabilityofanupmovement} = \frac{e^{r\Delta T} - d}{u - d}$$

Using the above values, the option price of final nodes are calculated and then we work backwards to calculate the option price of the first node. (Cox, Ross & Rubinstein, 1979; Rendleman & Barter, 1979; Lee, Lee, & Wei, 1991). The option price at a node is calculated using below equation,

$$\mathbf{C_o = e^{-r\Delta T}(pC_u + (1-p)C_d)}$$

Several Studies has been done in order to evaluate the consistency of Black-Scholes model as well as Binomial Tree Pricing model. This study is yet another among them which is constructed in a way to evaluate the consistency of those models by comparing them with the market prices and also cross examining the consistency through put-call parity. Put-call parity is a principle that defines the relationship between the price of European put options and European call options of the same class, that is, with the same underlying asset, strike price and expiration date(Stoll& Merton). Put-call parity states that simultaneously holding a short European put and long European call of the same class will deliver the same return as holding one forward contract on the same underlying asset, with the same expiration and a forward price equal to the option's strike price. If the prices of the put and call options diverge so that this relationship does not hold, an arbitrage opportunity exists, meaning that sophisticated traders can earn a theoretically risk-free profit. Such opportunities are uncommon and short-lived in liquid markets (Robert & Bruce 1979). In this study we have used put-call parity to ensure if an arbitrage opportunity exists from the calculated prices of option using both Black-Scholes Model and Binomial Tree Pricing model.

Theoretical Background

3.1. ANOVA

ANOVA is an abbreviation for the full name of the method: Analysis of Variance Invented by R.A. Fisher in the 1920's. The one-way Analysis of Variance can be used for the case of a quantitative outcome with a categorical explanatory variable that has two or more levels of treatment. The term one-way, also called one-factor, indicates that there is a single explanatory variable with two or more levels, and only one level of treatment is applied at any time for a given subject.

3.1.1. Assumptions

There are three main assumptions, listed here:

1. The dependent variable is normally distributed in each group that is being compared in the one-way ANOVA.
2. There is homogeneity of variances. This means that the population variances in each group are equal. This assumption can be testing by doing a Levene's test. If the Levene statistic is less than the critical value, the assumption is violated.
3. Independence of observations. This is mostly a study design issue and, as such, you will need to determine whether you believe it is possible that your observations are not independent based on your study design (e.g., group work/families/etc).

3.1.2. Methodology

A one-way analysis of variance is used when the data are divided into groups according to only one factor.

Assumption is made that the data $x_{11}, x_{12}, x_{13}, \ldots, x_{1n}$ are sample from population 1, $x_{21}, x_{22}, x_{23}, \ldots, x_{2n}$ are sample from population 2 likewise $x_{k1}, x_{k2}, x_{k3}, \ldots, x_{kn}$ are sample from population k. and x_{ij} isused todenote the data from the i^{th} group (level) and j^{th} observation.

The values of independent normal random variables X_{ij}, where i = 1, 2, 3…., kand j = 1, 2, 3…., n_i with mean μi and constant standard deviation σ, $X_{ij} \sim N(\mu i, \sigma)$.

Alternatively, each $X_{ij} = \mu i + \varepsilon ij$where εij are normally distributed independent random errors, $\varepsilon_{ij} \sim N(\mu, \sigma)$. Let $N = n_1 + n_2 + n_3 + \ldots + n_k$ is the total number of observations (the total sample size across all groups), where ni is sample size for the i^{th} group.The parameters of this model are the population means $\mu_1, \mu_2, \ldots \mu_k$ and the common standard deviation σ.

Using many separate two-sample t-tests to compare many pairs of means is a bad idea because it won't yield a p-value or a confidence level for the complete set of comparisons together.

In this method, the null hypothesis is tested

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k \qquad (1)$$

Against the alternative hypothesis

$$H_1: \exists 1 \leq i, l \leq k: \mu_i \neq \mu_l \qquad (2)$$

Let ¯(x_i)represent the mean sample i (i = 1,2, 3,…., k)

$$\overline{x_i} = \frac{1}{n_j} \sum_{j=1}^{n_i} x_i, j \qquad (3)$$

¯xrepresent the grand mean, the mean of all the data points:

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{k} \sum_{j=1}^{n_i} x_i, j \qquad (4)$$

s_i^2 represent the sample variance:

$$s_i^2 = \frac{1}{n_i - 1} \sum_{i=1}^{k} (x_{ij} - \bar{x}_i)^2 \qquad (4)$$

and s2 = MSE is an estimate of the variance σ2 common to all k populations,

$$s^2 = \frac{1}{N - k} \sum_{i=1}^{k} (n_i - 1) . s_i^2 \qquad (5)$$

ANOVA is centredon the idea to compare the variation between groups (levels) and the variation within samples by analyzing their variances.

The total sum of squares SST, sum of squares for error (or within groups) SSE, and the sum of squares for treatments (or between groups) SSC are given as follows

$$SST = \sum_{i=1}^{k} \sum_{j=1}^{n_i} (x_{ij}, \bar{x})^2 \qquad (6)$$

$$SSE = \sum_{i=1}^{k} \sum_{j=1}^{n_i} (x_{ij}, \bar{x})^2 = \sum_{i=1}^{k} (n_i - 1) . s_i^2 \qquad (7)$$

$$SSC = \sum_{i=1}^{k}\sum_{j=1}^{n_i}(\bar{x}_i - \bar{x})^2 = \sum_{i=1}^{k} n_i(\bar{x}_i - \bar{x})^2 \quad (8)$$

The deviation from an observation to the grand mean is considered in the following way:

$$x_{ij} - \bar{x} = (x_{ij} - \bar{x}_i) + (\bar{x}_i - \bar{x}) \quad (9)$$

Notice that the left side is at the heart of SST, and the right side has the analogous pieces of SSE and SSC. It actually works out that:

$$SST = SSE + SSC \quad (10)$$

The total mean sum of squares MST, the mean sums of squares for error MSE, and the mean sums of squares for treatment MSC are:

$$MST = \frac{SST}{df(SST)} = \frac{SST}{N-1} \quad (11)$$

$$MSE = \frac{SSE}{df(SSE)} = \frac{SSE}{N-k} \quad (12)$$

$$MSC = \frac{SSC}{df(SSC)} = \frac{SSC}{k-1} \quad (13)$$

The one-way ANOVA, assuming the test conditions are satisfied, uses the following test statistic:

$$F = \frac{MSC}{MSE} \quad (14)$$

Under H0 this statistic has Fisher's distribution F(k - 1 , N - k) . In case it holds for the test criteria

$$F > F_{1-\alpha,k-1,N-k} \quad (15)$$

Where F1-α,k-1,N-k is (1 – α) quantile of F distribution with k – 1 and N- k degrees of freedom, then hypothesis

H0 is rejected on significance level α and alternative hypothesis is accepted which implies that there is a significant differences between the samples studied. We have used ANOVA as a Statistical tool to determine whether there is any statistical difference between the calculated theoretical option prices with the actual option prices at the market.

3.2. T-TEST

A statistically significant t-test result is one in which a difference between two groups is unlikely to have occurred because the sample happened to be atypical. The one sample t-test is a statistical procedure used to determine whether a sample of observations could have been generated by a process with a specific mean. Statistical significance is determined by the size of the difference between the group averages, the sample size, and the standard deviations of the groups. For practical purposes statistical significance suggests that the two larger populations from which we sample are "actually" different. The one sample t-test makes several assumptions. Although t-tests are quite robust, it is good practice to evaluate the degree of deviation from these assumptions in order to assess the quality of the results. The one sample t-test has four main assumptions:

- The dependent variable must be continuous (interval/ratio).
- The observations are independent of one another.
- The dependent variable should be approximately normally distributed.
- The dependent variable should not contain any outliers.

The procedure for one sample t-test are first they calculate the mean and the standard deviation of the sample data that has been collocated and then they find the test statistic using the below formula

$$t = \frac{\bar{x} - \mu}{\sqrt{s^2/n}}$$

Where ¯xthe sample mean, s2 isis the sample variance, μ is the specified population mean and n is the sample size.

Once the t statistic is calculated, the probability of observing the test statistic under the null hypothesis is calculated. Statistical significance is determined by looking at the p-value. The p-value gives the probability of observing the test results under the null hypothesis. If the assumptions are true, smaller p-values indicate a result that is less likely to occur by chance. This also indicates decreased support for the null hypothesis, although this possibility can never be ruled out completely. The cutoff value at which statistical significance is claimed is decided on by the researcher but usually a value of .05 or less is chosen, ensuring approximately 95% confidence in the results.

We have used t-test in this paper to statistically test if there is any significant difference in the option prices calculated using the option pricing models and the actual market price.

Data and Methodology

4.1. Data:

The primary source of the data required is the National Stock Exchange (NSE) website. The Futures and Options (F&O) section of the NSE provides trading in options of 175 listed companies. The following data were collected for each of the 175 companies:

- The spot price of the underlying stock
- Three strike prices at which the options are trading
- Market prices of both call and put options at each of the strike prices

Thus, a total of 525 market prices of call and put options are collected from the NSE website. The data was collected on the period of 3rd February – 7th February, 2017. The expiry date for all the option contracts is 23rd February, 2017.

Further, the historical weekly closing prices of the 175 underlying companies were collected from Prowess IQ. The time frame of the historical data is from 3rd January 2015- 3rd February 2017. This data can be used to find the volatility of the underlying stock. The volatility is calculated as the standard deviation of the weekly return series which is then annualized over one year.

4.2. Methodology

The purpose of the study can be categorized into two parts as follows:

1. Comparison of the Pricing methods.
2. Evidence of Put-Call parity.

4.2.1. Comparison of Pricing Methods

The option prices are determined using Deriva Gem for both the Binomial Option Pricing Model (BOPM) and the Continuous Option Pricing Model (COPM). In both the models, the option prices can be calculated from the Spot price of the underlying (S), volatility (σ), Strike price (K), risk-free rate (r) and the time for the option to expire (t). The number of steps in the binomial tree (n) is taken as 5 for all the options.

Thus, a set of three prices, namely the market price; the BOPM price and the COPM price, are available for all the 525 different strike prices. To test the hypothesis that there is statistically no difference between the prices obtained from different methods, one way analysis of variance (ANOVA) technique is used. Statistical Packages for Social Science (SPSS) 20 is used to perform the one way ANOVA.

4.2.2. Evidence of Put-Call Parity

The put-call parity relationship is given by the equation:

$$c - p = S - Ke^{-rt}$$

Where c = price of call option for a strike price of an underlying, p = price of put option for the same strike price of the same underlying.

The equation can be rearranged as:

$$c - p - S + Ke^{-rt} = 0$$

The stock price S and the strike price K is constant for all pricing methods for a particular underlying stock. The call and put prices determined for each model can be used in the equation to test if put-call parity exists.

A one sample t test is be used to test our hypothesis that the put call parity holds true for all models. If the value of c-p determined from a particular model is statistically different from the value of S-Ke^(-rt), then that particular model does not prove the evidence of put-call parity.

Results

5.1. Comparison of the Pricing Methods Using One Way ANOVA

5.1.1 Comparison of Call prices

The call prices determined using the market, the BOPM and the COPM are compared using one way ANOVA. The table 1 shows the descriptive statistics for the call prices obtained using different pricing methods:

Conclusion

The Binomial Option Pricing Model and the Continuous Pricing Model still remain as the popular means of estimating option prices theoretically. This study is an attempt to check the soundness of these models in the current economic and financial environment. The market prices were compared with the theoretically estimated option prices and one way ANOVA was used to check if there is any difference between them. It has been found that both the call prices and put prices determined from the market as well as the theoretical models show no statistical difference between them. Therefore the market does not overstate or understate the prices from their theoretical value. The prices determined from each method were also checked for put-call parity condition. It has been found that the market prices violate the put-call parity relationship and hence arbitrage opportunities can be made possible by choosing the appropriate portfolio of securities. It has been further understood that the prices estimated using Binomial Option Pricing Model also violate put-call parity but the arbitrage opportunities are negligible. The prices calculated from the Continuous Option Pricing Model does not statistically violate the put-call parity condition and no arbitrage opportunities can be made possible. Thus it can be concluded that the continuous option pricing model still remains a valid model, even after two decades since its inception by Black and Scholes.

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Table 1: Descriptive Statistics for the call prices of the different option pricing methods.

Pricing method of call option	Number of observations	Minimum	Maximum	Mean	Median	Mode	Standard Deviation
Market Price	525	.05	1431.40	37.04	12.75	7.00	128.66
BOPM	525	.00	1449.46	36.89	11.77	.00	130.34
COPM	525	.00	1388.29	35.76	11.59	.00	124.54

Source: SPSS output

Before going for the ANOVA test, the homogeneity of variances assumption must be checked. The Levene's test can be used to serve this purpose.

Table 2: Levene's test for the call prices

Levene Statistic	Degree of freedom 1	Degree of freedom 2	Significance value
.022	2	1572	.978

Source: SPSS output

The significance value corresponding to the Levene's Statistic shown in the table 2 is higher than the p-value of 0.05. Therefore the population variances are assumed to be equal.

The one way ANOVA table for the call prices of market, BOPM and COPM are given in the table 3.

Table 3: One way ANOVA of call prices.

Source of variation	Sum of Squares	Degree of freedom	Mean Square	F	Significance value
Between Groups	515.347	2	257.674	.016	.984
Within Groups	25703138.050	1572	16350.597		
Total	25703653.397	1574			

Source: SPSS output

The significance value corresponding to the F-statistic is 0.984 which is higher the p-value of 0.05. Therefore the null hypothesis, that the call prices determined from three methods are equal, is not rejected.

5.1.2. Comparison of Put prices:

The descriptive statistics for the put prices obtained using different methods is given the table 4:

Table 4: Descriptive Statistics for the put prices of the different option pricing models

Pricing Method of Put Option	Number of observations	Minimum	Maximum	Mean	Median	Mode	Standard Deviation
Market Price	525	.05	1464.95	43.26	11.30	5.40	161.25
BOPM	525	.00	1318.95	33.78	10.11	.00	118.26
COPM	525	.00	1254.33	32.65	9.94	.00	112.46

Source: SPSS output

The Levene's test to check the homogeneity of variances among the put prices from different methods is given in the table 5.

Table 5: Levene's test for the put prices

Levene Statistic	Degree of freedom 1	Degree of freedom 2	Significance value
2.553	2	1572	.078

Source: SPSS output

The significance value of the Levene statistic is 0.078 which is greater than the p-value of 0.05. Therefore the homogeneity of variances assumption is not violated.

The one way ANOVA table for the put prices of market, BOPM, COPM are given in table 6.

Table 6: One way ANOVA of put prices.

Source of variation	Sum of Squares	Degrees of freedom	Mean Square	F	Significance value
Between Groups	35704.015	2	17852.007	1.018	.362
Within Groups	27580302.294	1572	17544.722		
Total	27616006.309	1574			

Source: SPSS output

The significance value for the ANOVA of put prices determined from different methods is 0.362 which is greater than 0.05. Therefore there is no statistical difference between the put prices determined from the three pricing methods.

5.2. Checking for Put-Call Parity:

The values of c-p-S+Ke^(-rt) for each strike price of the options are calculated and taken for analysis.
The table 7 shows the descriptive statistics for the values of c-p-S+Ke^(-rt) for all the three pricing methods.

Table 7: Descriptive Statistics for the values of c-p-S+Ke^(-rt) of different models

Pricing Method Used	Number of observations	Minimum	Maximum	Mean	Median	Mode	Standard Deviation
Market Price	525	-879.20	126.70	-9.34	-.12	-879.20	82.26
BOPM	525	-1.41E-004	1.55E-005	-1.34E-006	-6.43E-009	-1.41E-004	1.25E-005
COPM	525	-1.82E-012	3.64E-012	1.56E-014	0.00E+000	0.00E+000	2.68E-013

Source: SPSS output

The one sample T test for the evidence of put-call parity calculated using market prices, the BOPM prices and the COPM prices is summarized in table 8:

Table 8: One Sample T test of the values of c-p-S+Ke^(-rt) of different models

Pricing Method	T-statistic	Degree of freedom	Significance value (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
Market	-2.601	524	.010	-9.33658	-16.3891	-2.2841
BOPM	-2.448	524	.015	-1.31E-006	-2.41E-006	-2.64E-007
COPM	1.333	524	.183	1.53E-014	-7.38E-015	3.85E-014

Source: SPSS output

The significance value corresponding to the t-statistic of the market price is 0.01 which is less than 0.05. Therefore the market prices do not exhibit put-call parity and arbitrage opportunities are possible.
The significance value corresponding to the t-statistic of the BOPM is 0.015 which is less than 0.05. Therefore we can reject the hypothesis that the put-call parity exists. But looking at the confidence interval, the values of c-p-S+Ke^(-rt) are too low to provide any arbitrage opportunity.
The significance value corresponding to the t-statistic of the COPM is 0.183 which is less than 0.05. Therefore we can conclude that the prices of call and put calculated from the COPM exhibit put-call parity and there is no presence of arbitrage opportunities if the prices follow this model.