



Buoyancy and Elasticity of Tax Revenue: Measurement Techniques

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ABSTRACT

High degree of responsiveness of tax yield to changes in national income is a desirable characteristic of any tax system. If a government depends on tax revenue to finance a large part of its expenditure then it is necessary to ensure that an increasing proportion of national income flows into the public treasury as taxes. In other words, the requirement is to increase the incremental tax ratio. Computation of tax buoyancy and elasticity becomes relevant in this context. This paper explains the concepts of buoyancy and elasticity of tax revenue and describes the various methods to compute them.

Keywords: *Buoyancy, Elasticity, Tax yield.*

1.0 Automatic Response versus Discretionary Changes

Tax revenue may change through *automatic* response of the tax yield to changes in national income and/or through the imposition of new taxes, revision of the rates and/or the bases of the existing taxes, tax amnesties, stricter tax compliance and other administrative measures backed by legal action. Changes in the tax yield resulting from modifying such tax parameters (i.e. rate, base etc.) are called *discretionary changes* which are the result of legislative action.

With tax parameters held constant (i.e. discretionary changes being removed), automatic changes in the tax yield resulting from variations in the national income measure the elasticity or built-in-flexibility of a tax or a tax system. It is the ratio of percentage change in tax revenue (adjusted for discretionary changes) to the percentage change in national income. Changes in the tax yield flowing from the combined effect of automatic response and discretionary changes measure the buoyancy of a tax. It is computed by dividing the percentage change in tax yield by the percentage change in the national income.¹

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Buoyancy coefficient compares the actual growth of tax revenue with the growth in national income. It assesses the overall success of government measures to increase tax revenue. Elasticity coefficient, on the other hand, indicates the inherent responsiveness of a tax system to changes in national income. It reflects the revenue potential of the tax system.

If a tax system lacks elasticity, the government will have to make frequent changes in tax laws to boost revenue. This will cause legal complications, reduce administrative efficiency and prove to be politically inexpedient. Therefore, while casting the tax net, tax bases should be so selected and rate structure so designed that reasonable degree of elasticity is imparted to the tax system.

Buoyancy and elasticity attributes of a tax system assume varying importance during the course of development process. As economic activities pick up and monetisation spreads, more and more tax handles become available to the government. To extend and intensify the tax structure, new taxes are imposed, rates of existing taxes are revised, tax bases are widened and administrative machinery is toned up. These discretionary changes, which are part of the buoyancy concept, play the dominant role and in a sense initiate the process towards higher tax-to-national income ratio. When tax revenue forms quite a high ratio of national income, further legislative action becomes politically disadvantageous in view of tax payers' resistance.

Moreover, frequent changes in tax parameters create confusion and uncertainty harming the smooth growth of trade and industry. Thus, discretionary changes, though never coming to a halt in an expanding economy, lose significance in course of time. Elasticity of the tax system becomes more important at this stage. If tax system is sufficiently elastic, the process initiated by discretionary changes will be kept up automatically.

2.0 National Income versus Appropriate Tax Base

Should tax yield be related to national income or to the appropriate tax base? As early as 1937, Bretherton (1937, p. 171) in his classic article on the question of sensitivity of tax revenue (the first of its kind in public finance) observed, "Since money income is the ultimate source from which all taxes must be paid, it seems desirable to measure fluctuations in the yield of particular taxes against fluctuations in the total of national social income." In modern societies, most taxes are either based on, or in some ways related to, production, income, expenditure, wealth or transactions which tend to increase with the growth of national income. Hence, taxes should be so devised as to grow automatically with increase in national income.

Maxwell (1954, p. 105), however, holds the contrary view and maintains, "When

measurement of overall built-in-flexibility is in question, it seems proper to take national or gross national product as the base. But when particular taxes are being examined, it may seem appropriate to relate changes in tax yield to changes in the precise base upon which the tax is levied." In view of the current practice of relating tax yield to national income as well as to appropriate base, the problem of choosing the variables for comparison has considerably eased.²

If the tax system as such is related to national income, it is important to relate individual taxes also to national income to facilitate comparisons. If the elasticity of the tax system is 1 and that of a particular tax is less than 1, it is clear that the particular tax is a weakling of the system, the exact weakling influence being determined by its relative revenue significance in the tax system.

3.0 Disaggregation of Elasticity Coefficient

A tax system is composed of individual taxes. The overall elasticity of the system, though important may not provide the necessary information for policy purposes. It is, therefore, necessary to compute elasticities of individual taxes so that their relative influence on the overall elasticity may be ascertained. In the aggregate models, elasticity of tax revenue to national income is presented as a single number. Since the relative revenue significance of individual taxes (which constitute the tax system) is not the same as also their responses to changes in national income, it is more realistic to compute the elasticities of individual taxes separately to derive the overall tax elasticity. In fact, the overall elasticity is the weighted sum of elasticities of individual taxes—the weights being the proportional shares of individual taxes in total tax revenue³. This type of analysis permits identification of the sources of fast revenue growth or conversely the causes of lagging revenue growth. These relationships may be summarised symbolically as follows, where

T_i = Revenue from *ith* individual tax

B_i = Base of the *ith* individual tax

Y = National income

n = Number of taxes in the system

Then,

$$\text{Total tax revenue (T)} = \sum_{i=1}^n T_i$$

$$\text{Elasticity of total tax revenue to income } E_{TY} = \frac{\Delta T}{\Delta Y} \times \frac{Y}{T}$$

$$\text{Elasticity of } i\text{th individual tax to income } E_{TiY} = \frac{\Delta T_i}{\Delta Y} \times \frac{Y}{T_i}$$

In a system of n taxes:

$$E_{TY} = \frac{T_1}{T} \left(\frac{\Delta T_1}{\Delta Y} \times \frac{Y}{T_1} \right) + \frac{T_2}{T} \left(\frac{\Delta T_2}{\Delta Y} \times \frac{Y}{T_2} \right) + \dots + \frac{T_n}{T} \left(\frac{\Delta T_n}{\Delta Y} \times \frac{Y}{T_n} \right)$$

Or

$$E_{TY} = \sum_{i=1}^n \left(\frac{\Delta T_i}{T} \right) E_{TiY}$$

4.0 Decomposition of Elasticity Coefficient

Income elasticity of each individual tax may be further partitioned into two elements: elasticity of the tax to the base and the elasticity of the base to income. The percentage change in tax divided by the percentage change in base provides the coefficient of tax-to-base elasticity or rate response while percentage change in base divided by percentage change in national income signifies the elasticity of base-to-income or base response.

Provided the base and the income are perfectly related, the product of the elasticity of tax- to-base and the elasticity of base-to-national income would give exactly the same result as the direct measurement of tax-to-income elasticity⁴. Symbolically,

Elasticity of *ith* individual tax to its base

$$E_{TiY} = \frac{\Delta T_i}{T_i} \times \frac{B_i}{\Delta B_i}$$

Elasticity of *ith* individual tax base to income

$$E_{BiY} = \frac{\Delta B_i}{B_i} \times \frac{Y}{\Delta Y_i}$$

Elasticity of *ith* individual tax to income

$$E_{TiY} = \frac{\Delta T_i}{T_i} \times \frac{Y}{\Delta Y} = \left(\frac{\Delta T_i}{T_i} \times \frac{B_i}{\Delta B_i} \right) \left(\frac{\Delta B_i}{B_i} \times \frac{Y}{\Delta Y} \right)$$

To rewrite the equation

$$E_{TY} = \sum_{i=1}^n \left(\frac{T_i}{T} \right) E_{TiY} = \sum_{i=1}^n \left(\frac{T_i}{T} \right) (E_{TiBi}) (E_{BiY})$$

$$= \sum_{i=1}^n \left(\frac{T_i}{T} \right) \left(\frac{\Delta T_i}{T_i} \times \frac{B_i}{\Delta B_i} \right) \left(\frac{\Delta B_i}{B_i} \times \frac{Y}{\Delta Y} \right)$$

The above identity states that overall elasticity is the weighted sum of the product of the elasticity of tax-to-base and base-to-income for each individual tax—the weights being the proportional share of individual taxes in total tax revenue.

Such a decomposition of the elasticity of a tax into tax-to-base and base-to-income elasticities is more informative for policy purposes. High or low elasticity of a tax can better be explained by examining the tax-to-base and base-to-income elasticities. In other words, one can detect if it is the rate structure that is inherently regressive with respect to the base or it is the base that is proving sluggish. This information has direct relevance for policymakers because it identifies that part of revenue growth that lies within the control of the government. Thus, tax-to-base constituent of elasticity may be raised by improvement in administration, e.g. preventing tax evasion and adopting more efficient procedures. On the other hand, the growth of the tax base lies outside the control of the authorities (apart from the influence of tax policy itself) and is largely determined by the way in which the structure of the economy changes with economic growth (Mansfield, 1972).

5.0 Dissecting Discretionary Changes

Measurement of tax buoyancy does not pose any conceptual problem. However, computation of tax elasticity involves methodological difficulties. To measure elasticity coefficient, it is necessary to eliminate from the actual tax revenues the effects of discretionary changes. This problem is dealt with by two alternative methods, one precluding and the other requiring the reconstruction of actual tax revenue data to account for discretionary changes.

Use of Dummy Variables: Under the procedure that does not warrant reconstruction of historical tax revenue series, the most popular technique is the use of dummy variables⁵. This method requires the insertion of a dummy variable into the regression equation whenever a discretionary change occurs in the structure (i.e. rate and/or base) of a tax. A separate dummy variable is required for each year in which a legislative change occurs. An equation of the following form may be specified,

$$\log T = \log a + b \log Y + \sum_{i=1}^n (c_i D_i)$$

where D_i represents the i th year's dummy and n the number of years. This technique can be successfully used only when available time series is sufficient and the discretionary changes are few. Reliability of estimated coefficients will be greatly

reduced if (a) tax structure changes in two successive years, (b) changes are too frequent, (c) observations are limited and (d) changes occur in the last year of the sample period. To cope with the problem of frequent changes one may take into account only the important discretionary changes and apply dummy variables to each one of them.

Apart from dummy variable technique, some other procedures have also been suggested which require conversion of simple regression equation to multiple regression model to take account of discretionary changes. However, the practical applicability of these alternative techniques is doubtful in view of their drastic assumptions and stringent data requirements⁶. In fact, the choice of any method is conditioned more by the availability of required data in a country rather than the relative superiority of alternative statistical techniques.

Adjustments in Historical Tax Revenue Series: In the second set of procedures, the historical tax revenue series is subjected to adjustments to work out a hypothetical revenue series which reflects the revenue growth that would have taken place in the absence of legislative measures. Two alternative methods are followed to reconstruct the required simulated time series: (a) the constant tax structure method and (b) the proportional adjustment method.

Constant-rate-base Method: The constant-rate-base method requires the selection of a reference year and the application of tax rate and legally defined base of the reference year to other years. In other words, the first year of the study period is selected as the base year and that year's tax rate is applied to the size of the tax base (according to legal definition of the base in the first year) in different years to simulate revenue series to represent automatic growth. This procedure is based on the assumption that elasticity of demand, for products in the case of indirect taxes and for income in the case of income taxes, is zero. It is further assumed that changes in tax rates or in the tax base yield proportionate change in tax revenue. The practicability of this procedure will be severely curtailed if long-time series data on bases are not available as is generally the case in underdeveloped countries.

Proportional Adjustment Method: Under the proportional adjustment method, suitable adjustments to the actual revenue collected each year are made to work out the refined series. This method requires data on total tax yield and the estimates of the effects of discretionary tax changes in the year's receipts. This information is normally available in the budget papers of the government. The procedure for separating discretionary effects consists of two steps: (a) A preliminary series of adjusted tax receipts is constructed by subtracting from the actual yield for each year the estimates of the effects of discretionary changes in that year⁷. (b) The series thus obtained is further cleaned by applying a formula to segregate the continuing impact of each discretionary

action in subsequent years on the assumption that the original revenue effect of the change grew proportionately with the yield of the tax. The formula for computing cumulative effects of discretionary changes as given by Prest (1962) and developed by Mansfield (1972) is as follows:

$T_1, T_2 \dots T_t \dots T_n$ are actual tax yields for a series of years.

$D_1, \dots D_2 \dots D_t \dots D_n$ measure the effect of a discretionary change in the t th year on the t th year's revenue out-turn.

T_{ij} indicates the j th year's actual tax yield adjusted to the tax structure that existed in year i .

If $i = 1$ is the base year, the series $T_{11}, T_{12}, T_{13} \dots T_{1t} \dots T_{1n}$ represents the 'would have been tax yields' had the base year tax structure prevailed. The resultant refined series eliminates the effects of discretionary changes on the tax yields of the years following the base year. The final series may be computed as follows:

$$T_{11} = T_1$$

$$T_{12} = T_2 - D_2$$

$$T_{13} = T_{23} \times T_{12} / T_2$$

$$T_{14} = T_{34} \times T_{23} / T_3 \times T_{12} / T_2$$

$$T_{1j} = T_{j-1,j} \times T_{j-2,j-1} / T_{j-1} \dots T_{23} / T_3 \times T_{12} / T_2$$

In the above formula, the actual tax yield is multiplied by a sequence of multiplicative factors. "The effect of any one of these factors is to adjust tax yields to the tax structure that prevailed in the year to which the factor referred. Each such factor reveals what proportion of the total yield for that year would have accrued automatically in the absence of any discretionary changes for that year; the factor is found by dividing the actual tax yield (net of the discretionary effects of that year) by the total tax yield"⁸.

The proportional adjustment method is based on the assumption that the discretionary changes do not affect the automatic response of tax revenue through changes in relative prices. Another assumption is that the discretionary changes are no more or less progressive than the tax structure they modify. Also, it is possible that there is divergence between estimates and actuals of additional yields resulting from discretionary changes. If revenue effects are overestimated, the simulated series of automatic growth in revenue would be correspondingly underestimated and so also the elasticity coefficient. The opposite will hold good if revenue effects are underestimated.

Roy Bahl's Correction: To deal with the problem of discrepancies between the budget estimates of the revenue effects of discretionary changes and the actual realisation, Roy Bahl⁹ has suggested the following procedure which may be viewed as a logical extension of Prest's formulation.

- ^
- ΔT = Estimated increase in tax receipts
- ^
- ΔT_d = Estimated effect of the discretionary changes
- ΔT_a = Estimated automatic growth
- ΔT = Actual increase in tax revenues

Then, since $\Delta T = \Delta T_d + \Delta T_a$, and assuming a constancy of the ratio of discretionary changes to total tax change, the corrections for the discretionary changes will be

$$\Delta T_d = (\Delta T_d / \Delta T) \times \Delta T$$

This correction procedure is based on the assumption that the underestimate/overestimate of revenue would mainly be due to inaccurate forecasts of total income.

6.0 Estimation Techniques

Various measures are used to define the relationship between tax revenue and national income. The *marginal approach* merely utilises the ratio of the absolute change in tax revenue over a given period to the absolute change in national income over that same period, i.e.

$$M = \Delta T / \Delta Y$$

where M (marginal tax rate) is a measure of built-in-flexibility, and T and Y are tax revenue and national income respectively.

Another measure defines tax elasticity as the ratio of the percentage change in tax revenue to percentage change in national income over a given period i.e.

$$E = \frac{\Delta T}{T} \times \frac{Y}{\Delta Y} = \left(\frac{\Delta T}{\Delta Y} \times \frac{Y}{T} \right) = \left(\frac{\Delta T}{\Delta Y} / \frac{T}{Y} \right)$$

The relationship between the two measures stated above is clear. Elasticity is the ratio of the marginal tax rate ($\Delta T / \Delta Y$) to the average tax rate (T / Y). If marginal tax rate is more than the average tax rate, the tax elasticity will exceed unity.

Estimates of buoyancy/elasticity are obtained also by dividing the rate of growth (simple or compound) of the tax revenue by the rate of growth of national income or some other chosen macroeconomic aggregate.

The limitation with these methods is that they take into account the values of the

variables in the initial and terminal years only, i.e. year-to-year variations in tax yield are not reflected.

Alternatively, the use of regression technique is preferred because it takes into account each observed value of the two variables over the period of study. Estimation of elasticity/ buoyancy is obtained by fitting a log linear regression of tax revenue on national income/appropriate bases. The response coefficient from time series data is estimated by the use of the following revenue exponential function.

$$T = aY^b$$

or taking logarithms on both the sides,

$$\log T = \log a + b \log Y$$

In a least square fit of this logarithmically linear equation on time series data, the regression coefficient a denotes the level of the tax yield (T) when national income (Y) is zero. The regression coefficient b signifies the per cent change in (T) that accompanies 1 per cent change in (Y). If this be the equation for buoyancy estimation, the equation for elasticity may be specified as follows:

$$\log T' = \log a + b' \log Y$$

where T' is actual tax yield net of discretionary changes and b' the elasticity of tax revenue.

For responsiveness of tax-to-base and base-to-income, the expressions would be the following respectively:

$$\log T' = \log a + b \log B \text{ and}$$

$$\log B = \log a + b \log Y$$

where B is the base of the tax.

If coefficient b turns out to be more than 1, the responsiveness of tax system will be considered relatively high and if it is less than 1, the same will be termed as relatively low.

The above method assumes that the buoyancy/elasticity is constant over the range of income considered, i.e. proportionate response of the tax to an income change of 1 per cent is the same irrespective of the level of income. It also assumes the existence of a significant correlation between T and Y. An indication of this is provided by the statistic R^2 which measures the goodness of fit of the functional relationship being measured. In the absence of a significant correlation between the two variables, the least square estimate of b will convey little meaning.

Endnotes

1. If there are several discretionary changes of revenue significance during the reference period, the elasticity coefficient would be much lower than the buoyancy coefficient.

2. Since information on the true base of a tax is often inaccessible, proxy bases are often used.
3. Weight of a tax may be taken as revenue from that tax as a percentage of total tax revenue during the reference period.
4. The product of these two component elasticities would only approximate the estimate of tax to income elasticity because of errors in practical estimation. However, the closer the fit of the equation, the closer will be the approximation. In case R^2 is 1.00 the product of the elasticity of tax to base and base to income will be equal to the elasticity of tax to income.
5. N.M. Singer, 'The Use of Dummy Variables in Estimating the Income-elasticity of State Income-Tax Revenue', *National Tax Journal*, Volume 21, No. 2, June 1968, pp. 200-204. For a description of dummy variable methods, see, for example, R.J. Wonnacott and T.H. Wonnacott, *Econometrics*, (John Wiley & Sons, New York, 1970), Chapter 3, pp. 68-76.
6. For an explanation and review of these procedures, see V.G. Rao, *The Responsiveness of Tax System in India*, (New Delhi: Allied Publishers Pvt. Ltd., 1979), pp. 23-27.
7. Such actions may also affect demand of the concerned commodity and through that the revenue. It is, however, assumed that official estimates of revenue effect take into account these possibilities.
8. An alternative to this procedure suggested by Sahota asserts that the discretionary changes in any given year may affect the overall elasticity but in respect of tax yields they would influence only the yield of the year in which they have taken place. See, G.S. Sahota, *Indian Tax Structure and Economic Development*, (Asia Publishing House, Bombay, 1961), p. 7. This method may appear to be different from the one suggested by Prest. However, it has been demonstrated that the two methods give identical results. For a proof of this see R.J. Chelliah, and Sheetal Chand, 'A Note on Techniques of Adjusting the Tax Revenue Series for Discretionary Changes', *IMF Working Paper 1974* (Mimeographed).
9. R.W. Bahl, *Tax Revenue Forecasting in Developing Countries: A Conceptual Analysis*, IMF, DM/72/83, p. 13.

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